Di Ning

Quiz 2

VECTOR CALCULUS

MATH 21D, Sect 002, Winter Quarter, 2013

INSTRUCTOR: Blake Temple

22 students

1. (10pts) The integral and sum of integral in the following exercise give the area of region in the xy-plane. Sketch the region, label each bounding curve with its equation, and give the coordinates of the points where the curve intersect. Then find the area of the region.

$$\int_0^2 \int_{x^2-4}^0 dydx + \int_0^4 \int_0^{\sqrt{x}} dydx$$

2. (10pts) Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2+y^2)\, \cancel{dydx}\ dx\,dy$$

Solution:

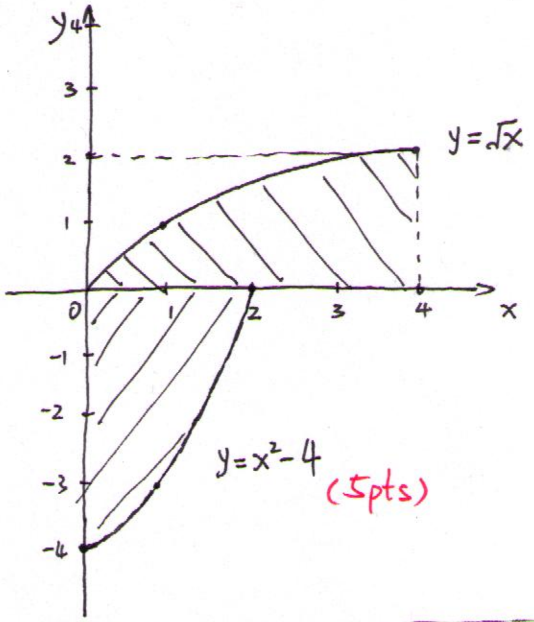
1. ① $0 \le x \le 2$

$x^2-4 \le y \le 0$

② $0 \le x \le 4$

$0 \le y \le \sqrt{x}$

$0 \le y^2 \le x$



(5pts)

$$\int_0^2\int_{x^2-4}^0 dy\,dx = \int_0^2 (-x^2+4)\,dx$$
$$= -\tfrac{1}{3}x^3+4x \Big|_0^2$$
$$= -\tfrac{8}{3}+8 = \tfrac{16}{3}$$

$$\int_0^4\int_0^{\sqrt{x}} dy\,dx = \int_0^4 (\sqrt{x})\,dx$$
$$= \tfrac{2}{3}x^{\frac{3}{2}}\Big|_0^4$$
$$= \tfrac{2}{3}\cdot 8$$

2. $$\int_{-1}^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$$

$$\boxed{= \int_0^{2\pi}\int_0^1 r^2\cdot r\,dr\,d\theta \quad \leftarrow (5pts)}$$

$$= \int_0^{2\pi} \left(\tfrac{1}{4}r^4\Big|_0^1\right) d\theta$$

$$= \tfrac{1}{4}\,\theta\Big|_0^{2\pi}$$

$$\boxed{= \tfrac{\pi}{2} \quad (5pts)}$$